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Instantons in four-Fermi term broken supersymmetric quantum mechanics with general potential

Agapitos Hatzinikitas and Ioannis Smyrnakis

University of Crete, Department of Applied Mathematics, L. Knosou-Ambelokipi, 71409 Iraklio, Crete, Greece

E-mail: ahatzini@tem.uoc.gr and smyrnaki@tem.uoc.gr

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Abstract

We have shown here how to find an integral representation for the solution of the Euclidean equations of motion of a quantum mechanical point particle in a general potential and in the presence of a four-Fermi term. The classical action in this theory depends explicitly on a set of four fermionic collective coordinates. The corrections to the classical action due to the presence of fermions are of topological nature in the sense that they depend only on the values of the fields at the boundary points $\tau \to \pm\infty$. As an application, the quantum mechanical sine-Gordon model with a four-Fermi term is solved explicitly and the corrections to the classical action are computed.

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1. Introduction

It is well known that the equations of motion for a point particle in Euclidean space moving under the influence of a Minkowski potential possessing at least two degenerate minima, admit finite action solutions which are the instantons. Their existence is responsible for tunnelling processes in the Minkowski space. The symmetries of the action are reflected in the space of instanton solutions. Local deformations of these solutions in the directions determined by the symmetries give rise to zero modes in the semiclassical expansion. To avoid infinities in the path integral due to these zero modes one is forced to introduce collective coordinates [1, 2].

In the presence of rigid supersymmetry the above solutions are still instantons provided that the fermions vanish [3, 4]. Applying the rigid supersymmetry transformation rules on these instantons one determines a more complete set of instantons where the fermions no longer vanish but instead they depend linearly on Grassmann collective coordinates (GCC). The number of GCC is equal to the number of Fermi fields present. Since the new instantons are related to the previous ones by supersymmetry, the classical action remains the same.

It is possible to break supersymmetry through the introduction of a four-Fermi term [5]. The new equations of motion can be solved iteratively starting with the instanton solution in the supersymmetric case. The iterative process terminates due to the nature of GCC, therefore it is possible to obtain exact solutions to the new equations of motion. Since there is no symmetry involved in obtaining these solutions the instanton action will change. The correction is the integral of a total derivative term, so it depends only on the boundary values of the fields. It depends also on the GCC introduced by supersymmetry. Note that the fermionic fields become infinite as $\tau \to \pm\infty$ while it is possible to keep the bosonic field finite by appropriate choice of the integration constants. Nevertheless, despite this infinity, the action remains finite.

As an explicit example, we consider the quantum mechanical sine-Gordon potential with a four-Fermi term. The iterative solution of the equations of motion is demonstrated explicitly determining the instantons in this way. The integration constant that renders the bosonic field finite is determined. In this case the instanton action becomes

$$S = -\frac{8m^3}{\lambda} + \epsilon^{ijkl}\xi_i\xi_j\xi_k\xi_l\frac{mg}{12}$$

where g, λ are coupling constants.

Finally, to obtain an integral expression for the solution of the equations of motion for a general potential, in terms of the bosonic part of the instanton when the Fermi field is zero, we make an appropriate change of variable to replace τ. It is interesting that in order to compute the action corrections it is not necessary to solve the nonlinear BPS equation [2]. In fact it is possible to express completely both the instanton and the finite boson integration constant in terms of the new variable.

2. The quantum mechanical model

We start by considering the following one-dimensional quantum mechanical model in Euclidean space:

$$S_{\text{cl}} = -\frac{1}{2}\int_{-\infty}^{\infty}[(\dot{x}(\tau))^2 + U^2(x(\tau))]\,\mathrm{d}\tau. \tag{1}$$

In writting this action we have performed Wick rotation to imaginary time. The potential $-\frac{1}{2}U^2(x)$ of the equivalent particle is assumed to have a number of degenerate minima.

The equation of motion is

$$\ddot{x} - U(x)U'(x) = 0. \tag{2}$$

The instanton solution of this equation satisfies the BPS equation [2]

$$\dot{x}_{\text{in}} + U(x_{\text{in}}) = 0 \tag{3}$$

subject to the conditions $x_{\text{in}}(\pm\infty) = C_\pm$, where $U(C_\pm) = 0$. The action is invariant under time translations which implies that if $x_{\text{in}}(\tau)$ is a solution to the BPS equation then $x_{\text{in}}(\tau - \tau_0)$ is also a solution. This means that

$$\tilde{Z}_0(\tau - \tau_0) = \dot{x}_{\text{in}}(\tau - \tau_0) = -\frac{\mathrm{d}}{\mathrm{d}\tau_0}x_{\text{in}}(\tau - \tau_0) \tag{4}$$

is a zero mode of the operator corresponding to the quadratic variation of the action around x_{in}. It satisfies the equation

$$\dot{\tilde{Z}}_0(\tau - \tau_0) + U'(x_{\text{in}})\tilde{Z}_0(\tau - \tau_0) = 0. \tag{5}$$

Note that the normalization of $\tilde{Z}_0$ defined as $\dot{x}_{\rm in}$ is just the absolute value of the classical action. This is so because

$$S_{\rm cl} = -\int_{-\infty}^{\infty} (\dot{x}_{\rm in})^2\,{\rm d}\tau = -\int_{-\infty}^{\infty} \tilde{Z}_0^2\,{\rm d}\tau \tag{6}$$

where we use the BPS equation. So, it is reasonable to define the normalized zero mode as follows:

$$Z_0 = |S_{\rm cl}|^{-1/2}\tilde{Z}_0. \tag{7}$$

It is possible to introduce fermions into this model by adding the terms

$$S_{\rm 2f} = -\frac{1}{2}\int_{-\infty}^{\infty} \left[\psi_i^T\dot{\psi}_i + \left(\psi_i^T\sigma_2\psi_i\right)U'\right]{\rm d}\tau \tag{8}$$

where $\sigma_2 = \begin{pmatrix} 0 & -{\rm i} \\ {\rm i} & 0 \end{pmatrix}$, ψ_i are two component Majorana fermions and the fermionic index is a colour index ranging in $i = 1, \ldots, 4$.

Each fermion is related to a boson through rigid $N = 1$ supersymmetry realized by the transformations

$$\delta x = \epsilon^T\sigma_2\psi \qquad \delta\psi = \sigma_2\dot{x}\epsilon - U\epsilon. \tag{9}$$

The spinor ϵ can be expanded in terms of the eigenstates $\psi_\pm = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ \pm{\rm i} \end{pmatrix}$ of σ_2 as follows:

$$\epsilon = \tfrac{1}{2}(\xi_+\psi_+ + \xi_-\psi_-) \tag{10}$$

and then it can be proved that if $\epsilon = \left(\frac{1+\sigma_2}{2}\right)\epsilon$ then,

$$\delta x = 0 \qquad \delta\psi = \xi_+\psi_+ Z_0(\tau). \tag{11}$$

Starting from the configuration $x = x_{\rm in}$, $\psi = 0$ and integrating over the above supersymmetry transformations we arrive at the instanton given by $x = x_{\rm in}$ and

$$\psi_i^{(1)} = \xi_i Z_0(\tau - \tau_0)\psi_+. \tag{12}$$

With each colour index we associate the same bosonic zero mode but different GCC ξ_i. Following [5], we add to the action a four-Fermi term which breaks the supersymmetry

$$S_{\rm 4f} = \frac{g}{4}\int_{-\infty}^{\infty} \epsilon_{ijkl}\left(\psi_i^T\sigma_1\psi_j\right)\left(\psi_k^T\sigma_1\psi_l\right){\rm d}\tau \tag{13}$$

where $\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$. This term makes sense only when we have four fermion colours. The new field equations are now

$$\ddot{x} - UU' = \tfrac{1}{2}\left(\psi_i^T\sigma_2\psi_i\right)U'' \tag{14}$$

$$\dot{\psi}_i + \sigma_2\psi_i U' = g\epsilon_{ijkl}\sigma_1\psi_j\left(\psi_k^T\sigma_1\psi_l\right). \tag{15}$$

Expanding ψ_i w.r.t the GCC one has $\psi_i = \psi_i^{(1)} + \psi_i^{(3)}$. Note that all the other terms in the expansion vanish. Making the ansatz

$$\psi_i^{(3)} = \alpha(\tau)\epsilon_{ijkl}\xi_j\xi_k\xi_l\psi_- \tag{16}$$

and plugging it into the fermionic field equation (15) one gets

$$\dot{\alpha} - \alpha U' = -gZ_0^3. \tag{17}$$

Since the solution of the homogeneous equation is Z_0^{-1} it is natural to set

$$\alpha(\tau) = Z_0^{-1}y(\tau). \tag{18}$$

This leads to the equation

$$\dot{y}(\tau) = -g Z_0^4. \tag{19}$$

Similarly, we can expand $x(\tau) = x_{\text{in}}(\tau) + x^{(4)}(\tau)$ and plug this into the bosonic field equation which gives

$$\ddot{x}^{(4)} - x^{(4)}(UU'' + U'^2)(x_{\text{in}}) = -\epsilon\xi^4 Z_0 \alpha(\tau) U''(x_{\text{in}}). \tag{20}$$

The solution of the homogeneous equation is $Z_0(\tau)$ so it is natural to set $x^{(4)} = \epsilon\xi^4 Z_0(\tau)\beta(\tau)$ where $\beta(\tau)$ satisfies

$$\frac{\mathrm{d}}{\mathrm{d}\tau}\big(Z_0^2\dot{\beta}\big) = -y(\tau) Z_0(\tau) U''(x_{\text{in}}) \tag{21}$$

and $\epsilon\xi^4 = \epsilon_{ijkl}\xi_i\xi_j\xi_k\xi_l$.

It is interesting to compute the corrections to the classical action due to $\psi_i^{(3)}$ and $x^{(4)}$. The two-Fermi term gives

$$\begin{aligned} S_{2\text{f}} &= -\frac{1}{2}\int_{-\infty}^{\infty} \big[\psi_i^{T(1)}\dot{\psi}_i^{(3)} + \psi_i^{T(3)}\dot{\psi}_i^{(1)} + \big(\psi_i^{T(1)}\sigma_2\psi_i^{(3)} + \psi_i^{T(3)}\sigma_2\psi_i^{(1)}\big)U'(x_{\text{in}})\big] \\ &= -\frac{1}{2}\int_{-\infty}^{\infty} \big[\psi_i^{T(3)}\big(\dot{\psi}_i^{(1)} + \sigma_2\psi_i^{(1)}U'\big) + \psi_i^{T(1)}\big(\dot{\psi}_i^{(3)} + \sigma_2\psi_i^{(3)}U'\big)\big] \\ &= -\frac{1}{2}(\epsilon\xi^4)\int_{-\infty}^{\infty} \dot{y}\,\mathrm{d}\tau = -\frac{1}{2}(\epsilon\xi^4)(y(\infty) - y(-\infty)) \end{aligned} \tag{22}$$

and the four-Fermi term

$$S_{4\text{f}} = \frac{g}{4}\int_{-\infty}^{\infty} \epsilon_{ijkl}\big(\psi_i^{T(1)}\sigma_1\psi_j^{(1)}\big)\big(\psi_k^{T(1)}\sigma_1\psi_l^{(1)}\big) = \frac{1}{4}(\epsilon\xi^4)(y(\infty) - y(-\infty)). \tag{23}$$

One easily checks that $S_{4\text{f}} = -\frac{1}{2}S_{2\text{f}}$.

The bosonic correction gives

$$\begin{aligned} S_{\text{bc}} &= -\int_{-\infty}^{\infty} [\dot{x}_{\text{in}}\dot{x}^{(4)} + x^{(4)}U(x_{\text{in}})U'(x_{\text{in}})] \\ &= -\int_{-\infty}^{\infty} \frac{\mathrm{d}}{\mathrm{d}\tau}(x^{(4)}\dot{x}_{\text{in}})\,\mathrm{d}\tau = -\epsilon\xi^4\sqrt{S_{\text{cl}}}\Big(\lim_{\tau\to\infty}\big(Z_0^2(\tau)\beta(\tau)\big) - \lim_{\tau\to-\infty}\big(Z_0^2(\tau)\beta(\tau)\big)\Big). \end{aligned} \tag{24}$$

It is worth noting that if the bosonic field is finite as $\tau = \pm\infty$ then $\lim_{\tau\to\infty}(\beta(\tau)Z_0(\tau))$ is finite and since $\lim_{\tau\to\infty} Z_0(\tau) = 0$ the bosonic correction vanishes.

3. The sine-Gordon model

For the sine-Gordon model the action is

$$S_{\text{cl}}^{\text{SG}} = -\frac{1}{2}\int_{-\infty}^{\infty}\left[\dot{x}^2 + \frac{2m^4}{\lambda}\left(1 - \cos\left(\frac{\sqrt{\lambda}}{m}x\right)\right)\right]\mathrm{d}\tau. \tag{25}$$

Here

$$U(x) = \frac{2m^2}{\sqrt{\lambda}}\sin\left(\frac{\sqrt{\lambda}}{2m}x\right). \tag{26}$$

The BPS equation takes the form

$$\dot{x}_{\text{in}} + \frac{2m^2}{\sqrt{\lambda}}\sin\frac{\sqrt{\lambda}}{2m}x_{\text{in}} = 0 \tag{27}$$

and can be easily solved to give

$$x_{\rm in}(\tau) = \pm 4 \frac{m}{\sqrt{\lambda}} \tan^{-1} \mathrm{e}^{-m(\tau-\tau_0)}. \tag{28}$$

The minus sign corresponds to the instanton solution and the plus sign to the anti-instanton. In what follows we are going to work with the instanton. The zero mode corresponding to the instanton is

$$\tilde{Z}_0(\tau) = \frac{\mathrm{d}x_{\rm in}}{\mathrm{d}\tau} = 2\frac{m^2}{\sqrt{\lambda}} \frac{1}{\cosh(m(\tau-\tau_0))} \tag{29}$$

and the classical action becomes

$$S_{\rm cl} = -\int_{-\infty}^{\infty} \tilde{Z}_0^2 \,\mathrm{d}\tau = -\frac{8m^3}{\lambda}. \tag{30}$$

So

$$Z_0 = \sqrt{\frac{m}{2}} \frac{1}{\cosh(m(\tau-\tau_0))}. \tag{31}$$

Upon introducing the two- and four-Fermi terms given in the previous section, we get that the fermionic field is $\psi_i = \psi_i^{(1)} + \psi_i^{(3)}$ where $\psi_i^{(1)}$ is given by (12) and $\psi_i^{(3)}$ is given by (16). The function $y(\tau)$ is the solution of the equation

$$\dot{y}(\tau) = -gZ_0^4(\tau) = -g\frac{m^2}{4} \frac{1}{\cosh^4(m(\tau-\tau_0))}. \tag{32}$$

This can be solved by setting $z = \tanh(m(\tau-\tau_0))$. With this change of variable the solution is written as

$$y(z) = -\frac{gm}{4}\left(a + z - \frac{1}{3}z^3\right) \tag{33}$$

and thus

$$\psi_i^{(3)} = -\frac{g}{2}\sqrt{\frac{m}{2}} \frac{1}{\sqrt{1-z^2}} \left(a + z - \frac{1}{3}z^3\right) \epsilon_{ijkl}\xi_j\xi_k\xi_l\psi_-. \tag{34}$$

To determine the form of $x^{(4)}$ we solve the bosonic field equation (21). In terms of the new variable z this equation becomes

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{\mathrm{d}\beta}{\mathrm{d}z} Z_0^4(\tau)\right) = \frac{1}{4}\sqrt{\frac{\lambda}{2m}}\, y(\tau). \tag{35}$$

This gives

$$\frac{\mathrm{d}\beta}{\mathrm{d}z} = -\frac{1}{4}\frac{g}{m}\sqrt{\frac{\lambda}{2m}} \left(\frac{A}{(1-z^2)^2} + \frac{\alpha z}{(1-z^2)^2} + \frac{z^2}{2(1-z^2)^2} - \frac{z^4}{12(1-z^2)^2}\right). \tag{36}$$

The solution to this equation is

$$\beta(z) = \frac{1}{4}\frac{g}{m}\sqrt{\frac{\lambda}{2m}} \left[\left(\frac{A}{2} + \frac{5}{24}\right)\frac{z}{z^2-1} - \left(\frac{A}{4} - \frac{1}{16}\right)\ln\left(\frac{z+1}{1-z}\right) + \frac{1}{12}z + \frac{1}{2}\frac{\alpha}{z^2-1} + B\right]. \tag{37}$$

The contribution to the two- and four-Fermi terms is

$$S_{2\rm f} + S_{4\rm f} = \frac{1}{2}S_{2\rm f} = \epsilon\xi^4 \frac{gm}{12}. \tag{38}$$

The bosonic correction is

$$S_{\rm bc} = -\epsilon\xi^4 \frac{1}{4} gm \left(A + \frac{5}{12}\right). \tag{39}$$

If the bosonic field is bounded at infinity then S_{bc} vanishes and this implies that $A = -\frac{5}{12}$. In this case the full action is

$$S_{\mathrm{tot}} = -\frac{8m^3}{\lambda} + \epsilon\xi^4 \frac{gm}{12}. \tag{40}$$

4. Generalization

It is worth mentioning that it is possible to find an integral representation for the solution of equations (19) and (21) if we change variables from τ to x_{in}. This is so because $\dot{x}_{\mathrm{in}} = \sqrt{|S_{\mathrm{cl}}|}Z_0$. Applying this change of variable to (19) we get

$$\frac{\mathrm{d}y}{\mathrm{d}x_{\mathrm{in}}} = -\frac{g}{\sqrt{|S_{\mathrm{cl}}|}} Z_0^3 = \frac{g}{|S_{\mathrm{cl}}|^2} U^3(x_{\mathrm{in}}) \tag{41}$$

where $|S_{\mathrm{cl}}| = \left| \int_{C_-}^{C_+} U(x_{\mathrm{in}})\,\mathrm{d}x_{\mathrm{in}} \right|$. This admits the solution

$$y = gm\tilde{\alpha} + \frac{g}{|S_{\mathrm{cl}}|^2} \int U^3(x_{\mathrm{in}})\,\mathrm{d}x_{\mathrm{in}}. \tag{42}$$

The integration constant $\tilde{\alpha}$ has been chosen to be dimensionless. Similarly equation (21) becomes

$$\frac{\mathrm{d}}{\mathrm{d}x_{\mathrm{in}}}\left(\frac{\mathrm{d}\beta}{\mathrm{d}x_{\mathrm{in}}} U^3(x_{\mathrm{in}})\right) = \sqrt{|S_{\mathrm{cl}}|}\, y(x_{\mathrm{in}}) U''(x_{\mathrm{in}}). \tag{43}$$

Integrating this equation once and using (41) we get

$$\frac{\mathrm{d}\beta}{\mathrm{d}x_{\mathrm{in}}} = \sqrt{|S_{\mathrm{cl}}|}\, y(x_{\mathrm{in}}) \frac{U'(x_{\mathrm{in}})}{U^3(x_{\mathrm{in}})} - \frac{g}{4} |S_{\mathrm{cl}}|^{-\frac{3}{2}} U(x_{\mathrm{in}}) + gm^2 \sqrt{|S_{\mathrm{cl}}|}\tilde{A} \frac{1}{U^3(x_{\mathrm{in}})}. \tag{44}$$

Integrating again we get

$$\begin{aligned}\beta(x_{\mathrm{in}}) = {} & -\frac{1}{2}|S_{\mathrm{cl}}|^{-\frac{3}{2}} \frac{1}{U^2(x_{\mathrm{in}})} \left(gm|S_{\mathrm{cl}}|^2\tilde{\alpha} + g\int U^3(x_{\mathrm{in}})\,\mathrm{d}x_{\mathrm{in}} \right) + \frac{1}{4} g |S_{\mathrm{cl}}|^{-\frac{3}{2}} \int U(x_{\mathrm{in}})\,\mathrm{d}x_{\mathrm{in}} \\ & + gm^2 |S_{\mathrm{cl}}|^{\frac{1}{2}} \tilde{A} \int \frac{1}{U^3(x_{\mathrm{in}})}\,\mathrm{d}x_{\mathrm{in}} + g|S_{\mathrm{cl}}|^{-\frac{1}{2}} \tilde{B}. \end{aligned} \tag{45}$$

Recall now that as $\tau \to \pm\infty$, $x_{\mathrm{in}} \to C_\pm$, where $U(C_\pm) = 0$. Demanding that the bosonic field correction is finite we get that $\lim_{x_{\mathrm{in}}\to C_\pm} (\beta(x_{\mathrm{in}})U(x_{\mathrm{in}}))$ is finite, so $\lim_{x_{\mathrm{in}}\to C_\pm} (\beta(x_{\mathrm{in}})U^2(x_{\mathrm{in}})) = 0$. This translates into the conditions

$$gm^2|S_{\mathrm{cl}}|^{\frac{1}{2}} \tilde{A} \lim_{x_{\mathrm{in}}\to C_+} \left(U^2(x_{\mathrm{in}}) \int_{x_0}^{x_{\mathrm{in}}} \frac{1}{U^3(s)}\,\mathrm{d}s \right) - \frac{g}{2}|S_{\mathrm{cl}}|^{-\frac{3}{2}} \int_{x_0}^{C_+} U^3(s)\,\mathrm{d}s = \frac{1}{2}\tilde{\alpha} gm |S_{\mathrm{cl}}|^{\frac{1}{2}} \tag{46}$$

$$gm^2|S_{\mathrm{cl}}|^{\frac{1}{2}} \tilde{A} \lim_{x_{\mathrm{in}}\to C_-} \left(U^2(x_{\mathrm{in}}) \int_{x_0}^{x_{\mathrm{in}}} \frac{1}{U^3(s)}\,\mathrm{d}s \right) - \frac{g}{2}|S_{\mathrm{cl}}|^{-\frac{3}{2}} \int_{x_0}^{C_-} U^3(s)\,\mathrm{d}s = \frac{1}{2}\tilde{\alpha} gm |S_{\mathrm{cl}}|^{\frac{1}{2}} \tag{47}$$

where the value of $\tilde{\alpha}$ is determined by the choice of x_0. Subtracting the two equations we arrive at

$$\tilde{A} = \frac{1}{2m^2}|S_{\mathrm{cl}}|^{-2} \frac{\int_{C_-}^{C_+} U^3(s)\,\mathrm{d}s}{(\lim_{x_{\mathrm{in}}\to C_+} - \lim_{x_{\mathrm{in}}\to C_-})\left(U^2(x_{\mathrm{in}}) \int_{x_0}^{x_{\mathrm{in}}} \frac{1}{U^3(s)}\,\mathrm{d}s\right)}. \tag{48}$$

These formulae have been checked in the cases of the double-well and the sine-Gordon potentials. For the double-well potential the results agree with those of [5] provided that we redefine our integration constants appropriately. In the case of the sine-Gordon model the results agree with those of the previous section under the following identification of the integration constants:

$$\tilde{A} = \frac{1}{16} - \frac{A}{4} \qquad \tilde{\alpha} = \frac{\alpha}{4} \qquad \tilde{B} = \frac{B}{2}. \tag{49}$$

5. Conclusion

We have determined the corrections to the supersymmetric instanton, for a quantum mechanical point particle in a general potential that admits at least two degenerate minima, due to the presence of a supersymmetry breaking four-Fermi term. Starting from the instanton solution of the supersymmetric case and applying an iterative procedure we obtain the exact solution of the new equations of motion. There is no symmetry involved in obtaining these solutions so the classical action will receive corrections. If we demand that the bosonic field remains finite as $\tau \to \pm\infty$ then only the two- and four-fermion terms contribute corrections to the classical action. The fermionic fields diverge as $\tau \to \pm\infty$; nevertheless, their corrections to the action remain finite. In the case of the sine-Gordon potential the classical action is modified by the contribution $\epsilon^{ijkl}\xi_i\xi_j\xi_k\xi_l\frac{mg}{12}$ where the ξ_i are fermionic collective coordinates. Finally, by a suitable change of variables, we determine the corrections to the classical action for a general potential and we compute the integration constant A that makes the bosonic field finite when $\tau \to \pm\infty$ in terms of the potential only.

References

[1] Rajaraman R 1982 *Solitons and Instantons* (Amsterdam: North-Holland)

[2] Shifman M A 1999 *Particle Physics and Field Theory (Lecture Notes in Physics vol 62)* (Singapore: World Scientific)

[3] Di Vecchia P and Ferrara S 1977 Classical solutions in two-dimensional supersymmetric field theories *Nucl. Phys.* B **130** 93

[4] Witten E and Olive D 1978 Supersymmetry algebras that include topological charges *Phys. Lett.* B **78** 97

[5] Vandoren S and van Nieuwenhuizen P 2001 New instantons in the double-well potential *Phys. Lett.* B **499** 280